L5 PFA SFA lavaan

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Generating Data

```r
set.seed(12345678)
library(mvtnorm)
# setting up matrices


npad       <- 100 #Occasions to throw out to wash away the effects of initial condition
time       <- 200 + npad
ne         <- 2 #Number of latent variables
ny         <- 6 #Number of manifest variables
psi        <- matrix(c(2.77, 2.47,   # Residual variance-covariance matrix
                       2.47, 8.40),
                     ncol = ne, byrow = T)
lambda     <- matrix(c(1, 0,         # Lambda matrix containing contemporaneous relations among
                       2, 0,         # observed variables and 2 latent variables.
                       1, 0,
                       0, 1,
                       0, 2,
                       0, 1),
                     ncol = ne, byrow = TRUE)
theta      <- diag(.5, ncol = ny, nrow = ny)  # Measurement error variances.
epsilon    <- rmvnorm(time, mean = c(0, 0, 0, 0, 0, 0), sigma = theta)  # Measurement errors.
beta       <- matrix(c(0.5, 0,     # Lagged directed relations among variables.
                       0.4, 0.5),
                     ncol = ne, byrow = TRUE)
zeta       <- mvtnorm::rmvnorm(time+npad, mean = c(0, 0), sigma = psi)   # Latent variable residuals.
etaC       <- matrix(0, nrow = ne, ncol = time + npad) # Set up matrix for contemporaneous variables.
etaL       <- matrix(0, nrow = ne, ncol = time + npad + 1) # Set up matrix for lagged variables.

etaL[,1]  <- c(0,0)
# generate factors
for (i in 1:(time+npad)){
  etaC[ ,i]   <- beta %*% etaL[ ,i] + zeta[i, ]
  etaL[ ,i+1] <- etaC[,i]
}
etaC <- etaC[,(npad+1):(npad+time)]
etaL <- etaL[,(npad+1):(npad+time)]
eta <- t(etaC)

# generate observed series
y    <- matrix(0, nrow = time, ncol = ny)
for (p in 1:nrow(y)){
  y[p, ] <- lambda %*% eta[p, ] + epsilon[p, ]
}

y <- y[101:time,]
```

Process Factor Analysis: lavaan

Prepare data

```r
# create block toeplitz data structure
y1   <- y[1:(nrow(y)-1), ]
y0   <- y[2:nrow(y), ]
y_bt <- data.frame(y1, y0)
names(y_bt) <- c(paste0("y", seq(1:6), "L"), paste0("y", seq(1:6), "C"))
```

CM5 (no cross-lag)

```
model5 <- '
eta1C =~ eq1*y1C + eq2*y2C + eq3*y3C
eta2C =~ eq4*y4C + eq5*y5C + eq6*y6C
eta1L =~ eq1*y1L + eq2*y2L + eq3*y3L
eta2L =~ eq4*y4L + eq5*y5L + eq6*y6L


eta1C ~~ eta2C
eta1L ~~ eta2L


eta1C ~ eta1L
eta2C ~ eta2L
'



fit  <- sem(model5, y_bt)
# summary(fit) # See full results from SEM.
lava <- parameterEstimates(fit)[ ,c(1, 2, 3, 5, 6,8)]
lava <- lava[lava$op %in% c("~", "=~"), ]
lava <- lava[,-2]
lava[,3:4] <- round(lava[,3:4], 3)
rownames(lava) <- NULL
knitr::kable(lava)
```

lhs	rhs	est	se	pvalue
eta1C	y1C	1.000	0.000	NA
eta1C	y2C	1.998	0.040	0
eta1C	y3C	1.029	0.026	0
eta2C	y4C	1.000	0.000	NA
eta2C	y5C	1.972	0.021	0
eta2C	y6C	0.993	0.013	0
eta1L	y1L	1.000	0.000	NA
eta1L	y2L	1.998	0.040	0
eta1L	y3L	1.029	0.026	0
eta2L	y4L	1.000	0.000	NA
eta2L	y5L	1.972	0.021	0
eta2L	y6L	0.993	0.013	0
eta1C	eta1L	0.464	0.055	0
eta2C	eta2L	0.556	0.052	0

```
round(fitMeasures(fit)[c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr", "bic")], digits = 3)
```

```
##   chisq      df  pvalue     cfi     tli   rmsea    srmr      bic
##  63.615  54.000   0.174   0.998   0.997   0.030   0.089 8461.752
```

3

CM 6 (incorrect cross-lag)

```
model6 <- '
eta1C =~ eq1*y1C + eq2*y2C + eq3*y3C
eta2C =~ eq4*y4C + eq5*y5C + eq6*y6C
eta1L =~ eq1*y1L + eq2*y2L + eq3*y3L
eta2L =~ eq4*y4L + eq5*y5L + eq6*y6L


eta1C ~~ eta2C
eta1L ~~ eta2L


eta1C ~ eta1L
eta2C ~ eta2L
eta1C ~ eta2L
'



fit  <- sem(model6, y_bt)
# summary(fit) # See full results from SEM.
lava <- parameterEstimates(fit)[ ,c(1, 2, 3, 5, 6,8)]
lava <- lava[lava$op %in% c("~", "=~"), ]
lava <- lava[,-2]
lava[,3:4] <- round(lava[,3:4], 3)
rownames(lava) <- NULL
knitr::kable(lava)
```

lhs	rhs	est	se	pvalue
eta1C	y1C	1.000	0.000	NA
eta1C	y2C	1.998	0.039	0.0000000
eta1C	y3C	1.029	0.026	0.0000000
eta2C	y4C	1.000	0.000	NA
eta2C	y5C	1.972	0.021	0.0000000
eta2C	y6C	0.993	0.013	0.0000000
eta1L	y1L	1.000	0.000	NA
eta1L	y2L	1.998	0.039	0.0000000
eta1L	y3L	1.029	0.026	0.0000000
eta2L	y4L	1.000	0.000	NA
eta2L	y5L	1.972	0.021	0.0000000
eta2L	y6L	0.993	0.013	0.0000000
eta1C	eta1L	0.396	0.068	0.0000000
eta2C	eta2L	0.591	0.057	0.0000000
eta1C	eta2L	0.065	0.040	0.1048756

```
round(fitMeasures(fit)[c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr", "bic")], digits = 3)
```

```
##   chisq      df  pvalue     cfi     tli   rmsea    srmr      bic
##  61.045  53.000   0.209   0.998   0.998   0.028   0.067 8464.474
```

4

Correct model (CM7)

```r
# create block toeplitz data structure
yL   <- y[1:(nrow(y)-1), ]
yC   <- y[2:nrow(y), ]
y_bt <- data.frame(yL, yC)
names(y_bt) <- c(paste0("y", seq(1:6), "L"), paste0("y", seq(1:6), "C"))
library(lavaan)
model <- '
eta1C =~ eq1*y1C + eq2*y2C + eq3*y3C
eta2C =~ eq4*y4C + eq5*y5C + eq6*y6C
eta1L =~ eq1*y1L + eq2*y2L + eq3*y3L
eta2L =~ eq4*y4L + eq5*y5L + eq6*y6L

eta1C ~~ eta2C
eta1L ~~ eta2L

eta1C ~ eta1L
eta2C ~ eta2L
eta2C ~ eta1L
'

fit  <- sem(model, y_bt)
# summary(fit) # See full results from SEM.
lava <- parameterEstimates(fit)[ ,c(1, 2, 3, 5, 6,8)]
lava <- lava[lava$op %in% c("~", "=~"), ]
lava <- lava[,-2]
lava[,3:4] <- round(lava[,3:4], 3)
rownames(lava) <- NULL
knitr::kable(lava)
```

lhs	rhs	est	se	pvalue
eta1C	y1C	1.000	0.000	NA
eta1C	y2C	2.000	0.039	0.00e+00
eta1C	y3C	1.029	0.026	0.00e+00
eta2C	y4C	1.000	0.000	NA
eta2C	y5C	1.972	0.021	0.00e+00
eta2C	y6C	0.993	0.013	0.00e+00
eta1L	y1L	1.000	0.000	NA
eta1L	y2L	2.000	0.039	0.00e+00
eta1L	y3L	1.029	0.026	0.00e+00
eta2L	y4L	1.000	0.000	NA
eta2L	y5L	1.972	0.021	0.00e+00
eta2L	y6L	0.993	0.013	0.00e+00
eta1C	eta1L	0.561	0.059	0.00e+00
eta2C	eta2L	0.396	0.062	0.00e+00
eta2C	eta1L	0.547	0.126	1.42e-05

```r
round(fitMeasures(fit)[c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr", "bic")], digits = 3)
```

```
   chisq      df  pvalue     cfi     tli   rmsea    srmr      bic
  45.461  53.000   0.759   1.000   1.002   0.000   0.013 8448.890
```

SFA

Lag one models

Prepare data

```r
# create block toeplitz data structure
y1   <- y[1:(nrow(y)-1), ]
y0   <- y[2:nrow(y), ]
y_bt <- data.frame(y1, y0)
names(y_bt) <- c(paste0("y", seq(1:6), "_1"), paste0("y", seq(1:6), "_0"))
```

CM1: 1 LV, 1 lag

lhs	rhs	est	se	pvalue
eta11	y1_1	1.000	0.000	NA
eta11	y2_1	2.008	0.200	0e+00
eta11	y3_1	1.081	0.108	0e+00
eta11	y4_1	2.901	0.214	0e+00
eta11	y5_1	5.723	0.419	0e+00
eta11	y6_1	2.876	0.212	0e+00
eta11	y1_0	0.616	0.118	2e-07
eta11	y2_0	1.167	0.223	2e-07
eta11	y3_0	0.613	0.121	4e-07
eta11	y4_0	1.715	0.209	0e+00
eta11	y5_0	3.382	0.407	0e+00
eta11	y6_0	1.725	0.208	0e+00
eta10	y1_0	1.000	0.000	NA
eta10	y2_0	2.008	0.200	0e+00
eta10	y3_0	1.081	0.108	0e+00
eta10	y4_0	2.901	0.214	0e+00
eta10	y5_0	5.723	0.419	0e+00
eta10	y6_0	2.876	0.212	0e+00

```
##   chisq      df  pvalue     cfi     tli   rmsea    srmr       bic
## 1390.921  53.000   0.000   0.708   0.637   0.356   0.185 9794.351
```

CM 3: 2 LVs, lag 1

```
model3 <- '

eta11 =~ 1*y1_1 + eq1*y2_1 + eq2*y3_1  +eq3*y1_0 + eq4*y2_0 + eq5*y3_0
eta10 =~ 1*y1_0 + eq1*y2_0 + eq2*y3_0

eta21 =~ 1*y4_1 + eq6*y5_1 + eq7*y6_1 + eq8*y4_0 + eq9*y5_0 + eq10*y6_0
eta20 =~ 1*y4_0 + eq6*y5_0 + eq7*y6_0


eta11 ~~ eta21 + 0*eta10 + 0*eta20
eta10 ~~ eta20 + 0*eta21

eta21 ~~ 0*eta20
'

fit  <- sem(model3, y_bt)
# summary(fit)
lava <- parameterEstimates(fit)[ ,c(1, 2, 3, 5, 6,8)]
lava <- lava[lava$op %in% c("~", "=~"), ]
lava <- lava[,-2]
lava[,3:4] <- round(lava[,3:4], 3)
rownames(lava) <- NULL
knitr::kable(lava)
```

lhs	rhs	est	se	pvalue
eta11	y1_1	1.000	0.000	NA
eta11	y2_1	2.006	0.043	0
eta11	y3_1	1.029	0.028	0
eta11	y1_0	0.473	0.060	0
eta11	y2_0	0.924	0.112	0
eta11	y3_0	0.488	0.062	0
eta10	y1_0	1.000	0.000	NA
eta10	y2_0	2.006	0.043	0
eta10	y3_0	1.029	0.028	0
eta21	y4_1	1.000	0.000	NA
eta21	y5_1	1.972	0.023	0
eta21	y6_1	0.991	0.014	0
eta21	y4_0	0.554	0.054	0
eta21	y5_0	1.092	0.103	0
eta21	y6_0	0.558	0.053	0
eta20	y4_0	1.000	0.000	NA
eta20	y5_0	1.972	0.023	0
eta20	y6_0	0.991	0.014	0

```
round(fitMeasures(fit)[c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr", "bic")], digits = 3)

##   chisq      df  pvalue     cfi     tli   rmsea    srmr     bic
## 62.879  50.000   0.104   0.997   0.996   0.036   0.088 8482.189
```

2 lags

Prep data

```
# create block toeplitz data structure
y2   <- y[1:(nrow(y)-2), ]
y1   <- y[2:(nrow(y)-1), ]
y0   <- y[3:nrow(y), ]
y_bt <- data.frame(y2, y1, y0)
names(y_bt) <- c(paste0("y", seq(1:6), "_2"), paste0("y", seq(1:6), "_1"), paste0("y", seq(1:6), "_0"))
```

CM 2: 1 LV, 2 lags

lhs	rhs	est	se	pvalue
eta12	y1_2	1.000	0.000	NA
eta12	y2_2	1.988	0.037	0.0e+00
eta12	y3_2	1.033	0.023	0.0e+00
eta12	y4_2	1.014	0.068	0.0e+00
eta12	y5_2	2.081	0.130	0.0e+00
eta12	y6_2	1.063	0.066	0.0e+00
eta12	y1_1	0.582	0.050	0.0e+00
eta12	y2_1	1.143	0.094	0.0e+00
eta12	y3_1	0.604	0.052	0.0e+00
eta12	y1_0	0.311	0.068	4.4e-06
eta12	y2_0	0.604	0.129	2.7e-06
eta12	y3_0	0.310	0.070	9.8e-06
eta12	y4_1	1.003	0.090	0.0e+00
eta12	y5_1	2.008	0.175	0.0e+00
eta12	y6_1	1.006	0.089	0.0e+00
eta12	y4_0	0.738	0.123	0.0e+00
eta12	y5_0	1.438	0.238	0.0e+00
eta12	y6_0	0.741	0.121	0.0e+00
eta11	y1_1	1.000	0.000	NA
eta11	y2_1	1.988	0.037	0.0e+00
eta11	y3_1	1.033	0.023	0.0e+00
eta11	y4_1	1.014	0.068	0.0e+00
eta11	y5_1	2.081	0.130	0.0e+00
eta11	y6_1	1.063	0.066	0.0e+00
eta11	y1_0	0.582	0.050	0.0e+00
eta11	y2_0	1.143	0.094	0.0e+00
eta11	y3_0	0.604	0.052	0.0e+00
eta11	y4_0	1.003	0.090	0.0e+00
eta11	y5_0	2.008	0.175	0.0e+00
eta11	y6_0	1.006	0.089	0.0e+00
eta10	y1_0	1.000	0.000	NA
eta10	y2_0	1.988	0.037	0.0e+00
eta10	y3_0	1.033	0.023	0.0e+00
eta10	y4_0	1.014	0.068	0.0e+00
eta10	y5_0	2.081	0.130	0.0e+00
eta10	y6_0	1.063	0.066	0.0e+00

```
##      chisq        df    pvalue        cfi       tli     rmsea      srmr
## 3380.248   133.000     0.000      0.530     0.459     0.351     0.149
##        bic
```

```
## 15804.211
```

CM4: 2 LVs, 2 lags

```
# create block toeplitz data structure

model4 <- '
eta12 =~ 1*y1_2 + eq1*y2_2 + eq2*y3_2 + eq3*y1_1 + eq4*y2_1 + eq5*y3_1 + y1_0 + y2_0 + y3_0
eta11 =~ 1*y1_1 + eq1*y2_1 + eq2*y3_1 + eq3*y1_0 + eq4*y2_0 + eq5*y3_0
eta10 =~ 1*y1_0 + eq1*y2_0 + eq2*y3_0


eta22 =~ 1*y4_2 + eq6*y5_2 + eq7*y6_2 + eq8*y4_1 + eq9*y5_1 + eq10*y6_1 + y4_0 + y5_0 + y6_0
eta21 =~ 1*y4_1 + eq6*y5_1 + eq7*y6_1 + eq8*y4_0 + eq9*y5_0 + eq10*y6_0
eta20 =~ 1*y4_0 + eq6*y5_0 + eq7*y6_0


eta12 ~~ eta22 + 0*eta11 + 0*eta10 + 0*eta21 + 0*eta20
eta11 ~~ eta21 + 0*eta10 + 0*eta20 + 0*eta22
eta20 ~~ 0*eta22
eta21 ~~ 0*eta20 + eta22
eta10 ~~ eta20 + 0*eta21 + 0*eta22
'


fit  <- sem(model4, y_bt)
summary(fit)
```

lavaan (0.5-23.1097) converged normally after 165 iterations

  Number of observations   198

  Estimator   ML
  Minimum Function Test Statistic   138.934
  Degrees of freedom   127
  P-value (Chi-square)   0.221

Parameter Estimates:

  Information   Expected
  Standard Errors   Standard

Latent Variables:
                   Estimate Std.Err z-value P(>|z|)
  eta12 =~
    y1_2   1.000
    y2_2   (eq1)   2.012   0.036   55.717   0.000
    y3_2   (eq2)   1.029   0.024   43.523   0.000
    y1_1   (eq3)   0.506   0.048   10.643   0.000
    y2_1   (eq4)   0.990   0.089   11.150   0.000
    y3_1   (eq5)   0.525   0.049   10.698   0.000
    y1_0   0.261   0.064   4.092   0.000
    y2_0   0.509   0.120   4.254   0.000
    y3_0   0.261   0.066   3.951   0.000
  eta11 =~
    y1_1   1.000
    y2_1   (eq1)   2.012   0.036   55.717   0.000
    y3_1   (eq2)   1.029   0.024   43.523   0.000

```
    y1_0     (eq3)    0.506    0.048    10.643    0.000
    y2_0     (eq4)    0.990    0.089    11.150    0.000
    y3_0     (eq5)    0.525    0.049    10.698    0.000
  eta10 =~
    y1_0              1.000
    y2_0     (eq1)    2.012    0.036    55.717    0.000
    y3_0     (eq2)    1.029    0.024    43.523    0.000
  eta22 =~
    y4_2              1.000
    y5_2     (eq6)    1.974    0.019   101.299    0.000
    y6_2     (eq7)    0.990    0.012    84.850    0.000
    y4_1     (eq8)    0.644    0.053    12.060    0.000
    y5_1     (eq9)    1.273    0.104    12.226    0.000
    y6_1    (eq10)    0.655    0.053    12.404    0.000
    y4_0              0.416    0.068     6.145    0.000
    y5_0              0.804    0.132     6.103    0.000
    y6_0              0.396    0.067     5.887    0.000
  eta21 =~
    y4_1              1.000
    y5_1     (eq6)    1.974    0.019   101.299    0.000
    y6_1     (eq7)    0.990    0.012    84.850    0.000
    y4_0     (eq8)    0.644    0.053    12.060    0.000
    y5_0     (eq9)    1.273    0.104    12.226    0.000
    y6_0    (eq10)    0.655    0.053    12.404    0.000
  eta20 =~
    y4_0              1.000
    y5_0     (eq6)    1.974    0.019   101.299    0.000
    y6_0     (eq7)    0.990    0.012    84.850    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  eta12 ~~
    eta22             5.257    0.686     7.666    0.000
    eta11             0.000
    eta10             0.000
    eta21             0.000
    eta20             0.000
  eta11 ~~
    eta21             2.502    0.405     6.182    0.000
    eta10             0.000
    eta20             0.000
    eta22             0.000
  eta22 ~~
    eta20             0.000
  eta21 ~~
    eta20             0.000
  eta22 ~~
    eta21            -2.148    0.745    -2.884    0.004
  eta10 ~~
    eta20             2.450    0.411     5.958    0.000
    eta21             0.000
    eta22             0.000

Variances:
```

	Estimate	Std.Err	z-value	P(>|z|)
.y1_2	0.470	0.059	7.910	0.000
.y2_2	0.279	0.145	1.923	0.055
.y3_2	0.561	0.068	8.224	0.000
.y1_1	0.493	0.062	8.006	0.000
.y2_1	0.313	0.146	2.136	0.033
.y3_1	0.553	0.068	8.150	0.000
.y1_0	0.499	0.063	7.965	0.000
.y2_0	0.287	0.151	1.897	0.058
.y3_0	0.563	0.069	8.134	0.000
.y4_2	0.486	0.063	7.715	0.000
.y5_2	0.515	0.162	3.185	0.001
.y6_2	0.415	0.057	7.280	0.000
.y4_1	0.485	0.063	7.727	0.000
.y5_1	0.547	0.161	3.400	0.001
.y6_1	0.409	0.056	7.261	0.000
.y4_0	0.489	0.063	7.752	0.000
.y5_0	0.515	0.161	3.202	0.001
.y6_0	0.406	0.056	7.225	0.000
eta12	4.231	0.450	9.406	0.000
eta11	2.862	0.308	9.308	0.000
eta10	2.839	0.305	9.306	0.000
eta22	15.173	1.536	9.877	0.000
eta21	9.211	0.961	9.581	0.000
eta20	9.205	0.948	9.706	0.000

```r
lava <- parameterEstimates(fit)[ ,c(1, 2, 3, 5, 6,8)]
lava <- lava[lava$op %in% c("~", "=~"), ]
lava <- lava[,-2]
lava[,3:4] <- round(lava[,3:4], 3)
rownames(lava) <- NULL
knitr::kable(lava)
```

lhs	rhs	est	se	pvalue
eta12	y1_2	1.000	0.000	NA
eta12	y2_2	2.012	0.036	0.00e+00
eta12	y3_2	1.029	0.024	0.00e+00
eta12	y1_1	0.506	0.048	0.00e+00
eta12	y2_1	0.990	0.089	0.00e+00
eta12	y3_1	0.525	0.049	0.00e+00
eta12	y1_0	0.261	0.064	4.28e-05
eta12	y2_0	0.509	0.120	2.10e-05
eta12	y3_0	0.261	0.066	7.78e-05
eta11	y1_1	1.000	0.000	NA
eta11	y2_1	2.012	0.036	0.00e+00
eta11	y3_1	1.029	0.024	0.00e+00
eta11	y1_0	0.506	0.048	0.00e+00
eta11	y2_0	0.990	0.089	0.00e+00
eta11	y3_0	0.525	0.049	0.00e+00
eta10	y1_0	1.000	0.000	NA
eta10	y2_0	2.012	0.036	0.00e+00
eta10	y3_0	1.029	0.024	0.00e+00
eta22	y4_2	1.000	0.000	NA
eta22	y5_2	1.974	0.019	0.00e+00

lhs	rhs	est	se	pvalue
eta22	y6_2	0.990	0.012	0.00e+00
eta22	y4_1	0.644	0.053	0.00e+00
eta22	y5_1	1.273	0.104	0.00e+00
eta22	y6_1	0.655	0.053	0.00e+00
eta22	y4_0	0.416	0.068	0.00e+00
eta22	y5_0	0.804	0.132	0.00e+00
eta22	y6_0	0.396	0.067	0.00e+00
eta21	y4_1	1.000	0.000	NA
eta21	y5_1	1.974	0.019	0.00e+00
eta21	y6_1	0.990	0.012	0.00e+00
eta21	y4_0	0.644	0.053	0.00e+00
eta21	y5_0	1.273	0.104	0.00e+00
eta21	y6_0	0.655	0.053	0.00e+00
eta20	y4_0	1.000	0.000	NA
eta20	y5_0	1.974	0.019	0.00e+00
eta20	y6_0	0.990	0.012	0.00e+00

```r
round(fitMeasures(fit)[c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr", "bic")], digits = 3)
```

```
   chisq        df    pvalue       cfi       tli     rmsea      srmr
 138.934   127.000     0.221     0.998     0.998     0.022     0.084
     bic
12594.627
```